## 3.23 Powers and Roots of Complex Numbers

**De Moivre's Theorem**

If $z = r(\cos\theta + i\sin\theta)$ is a complex number and $n$ is any positive integer, then

$$z^n = r^n\left(\cos n\theta + i\sin n\theta\right)$$

**Examples:**

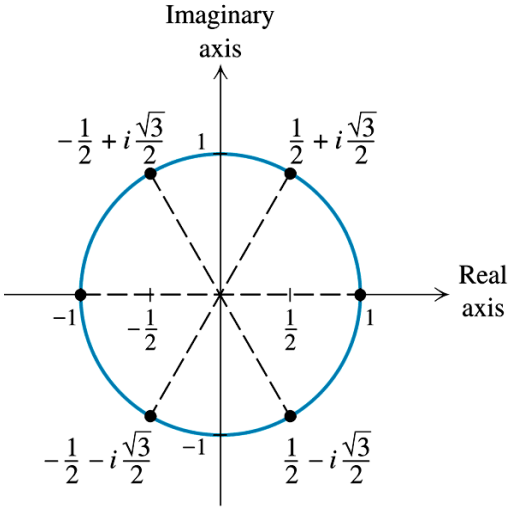
a) Simplify $(1+i)^6$.

b) Simplify $\left(\sqrt{3}-i\right)^4$.

**Roots of a Complex Number**

How many square roots does 4 have?
How many square roots does $-9$ have?
How many sixth roots does 1 have?

It turns out that 1 has 6 sixth roots, and they are distributed evenly around the complex plane.




Sixth Roots of Unity

The complex number $a+bi$ is an $n$th root of the complex number $z$ if $\left(a+bi\right)^n = z$.

For any positive integer $n$, the complex number $z = r(\cos\theta + i\sin\theta)$ has exactly $n$ distinct $n$th roots given by the expression $r^{1/n}\left[\cos\left(\frac{\theta+360°k}{n}\right)+i\sin\left(\frac{\theta+360°k}{n}\right)\right]$ for $k = 0,1,2,\ldots,n-1$.

In radians, the roots are given by $r^{1/n}\left[\cos\left(\frac{\theta+2k\pi}{n}\right)+i\sin\left(\frac{\theta+2k\pi}{n}\right)\right]$ for $k = 0,1,2,\ldots,n-1$.

The first of the $n$ roots has an argument of $\frac{\theta}{n}$, and the other roots are spaced $\left(\frac{360}{n}\right)^\circ$ apart.

(The circle is divided evenly into $n$ pieces.)

**Examples:**

a) Find all of the fourth roots of the complex number $-8-8i\sqrt{3}$.

b) Find all the cube roots of 125.

c) Find all complex solutions to $x^4-81=0$.